Moments of the photon spectrum in the inclusive $B \to X_s \gamma$ decay*

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Abstract

The moments of the photon spectrum in the inclusive $B \to X_s \gamma$ decay can be calculated to order α_s accuracy at present, without knowing the α_s corrections to the effective Hamiltonian. We discuss the standard model predictions, and how the moments of the photon spectrum are related to certain matrix elements of the heavy quark effective theory. The sensitivity of these moments to new physics is small, and they provide a model-independent determination of the b quark pole mass (at order α_s), or equivalently, the matrix elements $\bar{\Lambda}$ and λ_1 of the effective theory.

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I. INTRODUCTION

The inclusive $B \rightarrow X_s \gamma$ decay has received a lot of attention in recent years [1–6], primarily due to its sensitivity to physics beyond the standard model (SM) [2,5,6]. As any flavor changing neutral current process, it can only arise at one-loop level in the SM, and therefore possible new physics can yield comparable contributions. However, the recent CLEO measurement [7] excludes large deviations from the SM.

Since the b quark is heavy compared to the QCD scale, the inclusive $B \rightarrow X_s \gamma$ decay rate can be calculated in a systematic QCD-based expansion [8]. The decay rate computed in the $m_b \rightarrow \infty$ limit coincides with the free quark decay result. Corrections can then be included in an expansion in powers of $1/m_b$ and $\alpha_s(m_b)$.

At present, the theoretical prediction for the decay rate suffers from large uncertainties, as the result is only known in the leading logarithmic approximation. To refine the theoretical prediction and thus increase the sensitivity to new physics, a next-to-leading order calculation is needed. That is a very demanding task, as it requires the evaluation of many two-loop and even the infinite parts of three-loop diagrams. In the absence of such a calculation, and since the recent CLEO result shows no evidence for new physics contributing significantly to the $B \rightarrow X_s \gamma$ decay, we investigate what the presently available measurement could teach us.

We point out that the moments of the photon spectrum can be obtained to order α_s accuracy by a relatively simple calculation. We evaluate these corrections to the first few moments of the photon spectrum. Since $B \rightarrow X_s \gamma$ is a two-body decay at the quark level (at leading order in α_s), the photon spectrum is monochromatic in the spectator model. Therefore, the moments are also sensitive to the nonperturbative corrections in the heavy quark expansion. They provide a model-independent determination of the b quark pole mass, *i.e.*, the matrix elements $\bar{\Lambda}$ and λ_1 of the heavy quark effective theory (HQET).

II. THE EFFECTIVE HAMILTONIAN

The $B \to X_s \gamma$ decay in the standard model is mediated by penguin diagrams. The QCD corrections to this process form a power series in the parameter $\alpha_s \ln(M_W^2/m_b^2)$, that is too large to provide a reliable expansion. Therefore, it is convenient to integrate out the virtual top quark and W boson effects (and possible new physics) at the W scale, and sum up the large logarithms using the operator product expansion and the renormalization group. We work with the operator basis and effective Hamiltonian of Ref. [2]

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) \, O_i(\mu) \,, \tag{2.1}$$

where

$$
\begin{aligned}
O_1 &= (\bar{c}_{L\beta} \gamma^\mu b_{L\alpha}) (\bar{s}_{L\alpha} \gamma_\mu c_{L\beta}) \,, & O_2 &= (\bar{c}_{L\alpha} \gamma^\mu b_{L\alpha}) (\bar{s}_{L\beta} \gamma_\mu c_{L\beta}) \,, \\
O_7 &= \frac{e}{16\pi^2} m_b \, \bar{s}_{L\alpha} \, \sigma^{\mu\nu} F_{\mu\nu} \, b_{R\alpha} \,, & O_8 &= \frac{g}{16\pi^2} m_b \, \bar{s}_{L\alpha} \, \sigma^{\mu\nu} T_{\alpha\beta}^a \, G_{\mu\nu}^a \, b_{R\beta} \,.
\end{aligned}
\tag{2.2}
$$

We listed here only the operators whose Wilson coefficients are of order unity; $C_3 - C_6$ are about an order of magnitude smaller, since they arise only due to operator mixing. As the matrix elements of all operators except O_7 contain an overall factor of α_s (once we use the "effective" Wilson coefficients [5] defined below), we shall neglect $O_3 - O_6$.

The order α_s corrections to the the photon spectrum come from three sources [5]:

(i) Corrections to $C_i(M_W)$ coming from the matching of the SM matrix elements onto the effective Hamiltonian: the order α_s corrections to $C_7(M_W)$ and $C_8(M_W)$ are not yet known.

(ii) Corrections to the running of the Wilson coefficients C_i between the W and the m_b scale: the next-to-leading order mixing of the dimension six with the dimension five operators resulting from three-loop diagrams is not known at present.

(iii) The α_s corrections to the matrix elements of the operators in the effective Hamiltonian at the low scale: these corrections also involve unknown two-loop diagrams.

Certain parts of the next-to-leading log anomalous dimension matrix (ii) have been calculated [4]. The unknown contributions are expected to be significant, as these terms should

reduce the large μ-dependence of the leading log result. The last class of corrections (iii) has been considered in Ref. [9]. However, important two-loop diagrams involving O_2, that contribute to the spectrum near the maximal photon energy, were neglected. Moreover, in the absence of the next-to-leading order result for $C_7(\mu)$, any calculation of the spectrum inevitably contains renormalization scheme dependence at order α_s, and is therefore ill-defined.[*]

Since none of these three ingredients of the full order α_s calculation are known, it may be surprising that the moments can be calculated to this accuracy. We prove this in the next section. Thus, for our purposes it is sufficient to summarize the results for the coefficients of the effective Hamiltonian in the leading logarithmic approximation, for which we adopt the scheme independent definitions of [5]. In the standard model $C_2(M_W) = 1$, and

$$
\begin{aligned}
C_7(M_W) &= \frac{3x^3 - 2x^2}{4(x-1)^4} \ln x + \frac{-8x^3 - 5x^2 + 7x}{24(x-1)^3} \,, \\
C_8(M_W) &= \frac{-3x^2}{4(x-1)^4} \ln x + \frac{-x^3 + 5x^2 + 2x}{8(x-1)^3} \,,
\end{aligned}
\tag{2.3}
$$

where $x = m_t^2 / M_W^2$. At a low scale μ, these coefficients become

$$
\begin{aligned}
C_2(\mu) &= \frac{1}{2} \left(\eta^{6/23} + \eta^{-12/23} \right) C_2(M_W) \,, \\
C_7^{\rm eff}(\mu) &= \eta^{16/23} C_7(M_W) + \frac{8}{3} \left(\eta^{14/23} - \eta^{16/23} \right) C_8(M_W) + C_2(M_W) \sum_{i=1}^{8} h_i \, \eta^{a_i} \,, \\
C_8^{\rm eff}(\mu) &= \eta^{14/23} C_8(M_W) + C_2(M_W) \sum_{i=1}^{5} g_i \, \eta^{b_i} \,,
\end{aligned}
\tag{2.4}
$$

where $\eta = \alpha_s(M_W)/\alpha_s(\mu)$, and the numerical values of the h_i's, g_i's, a_i's, and b_i's can be found in [5]. The scale is usually chosen to be $\mu = m_b$, and one estimates the uncertainties related to the unknown higher order terms by varying μ, typically between $m_b/2$ and $2m_b$. However, our results will be scale independent to order α_s, so this is not going to be a large uncertainty.

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[*]We thank Mark Wise for pointing this out to us.

III. MOMENTS

In this section we show that moments of the photon spectrum can be calculated to order α_s, although none of the three previously described ingredients of the full order α_s computation of the decay rate are known. Since experimentally one needs to make a lower cut on the photon energy, we define the moments of the photon spectrum as

$$
M_n(E_0) = \frac{\displaystyle\int_{E_0}^{E_\gamma^{\mathrm{max}}} E_\gamma^n \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma}\,\mathrm{d}E_\gamma}{\displaystyle\int_{E_0}^{E_\gamma^{\mathrm{max}}} \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma}\,\mathrm{d}E_\gamma}\,.
\tag{3.1}
$$

Here $E_\gamma^{\mathrm{max}} = [M_B^2 - (M_K + M_\pi)^2]/2M_B$ is the maximal possible photon energy. To illustrate our argument, we denote schematically the contribution of a given operator O_i to the n-th moment of the spectrum by $\langle O_i \rangle_n$. Then we can rewrite the moments M_n as

$$
M_n \sim \left| \frac{C_7^{\mathrm{eff}} \langle O_7 \rangle_n + C_8^{\mathrm{eff}} \langle O_8 \rangle_n + C_2 \langle O_2 \rangle_n}{C_7^{\mathrm{eff}} \langle O_7 \rangle_0 + C_8^{\mathrm{eff}} \langle O_8 \rangle_0 + C_2 \langle O_2 \rangle_0} \right|^2 = \left| \frac{\langle O_7 \rangle_n + \dfrac{C_8^{\mathrm{eff}}}{C_7^{\mathrm{eff}}} \langle O_8 \rangle_n + \dfrac{C_2}{C_7^{\mathrm{eff}}} \langle O_2 \rangle_n}{\langle O_7 \rangle_0 + \dfrac{C_8^{\mathrm{eff}}}{C_7^{\mathrm{eff}}} \langle O_8 \rangle_0 + \dfrac{C_2}{C_7^{\mathrm{eff}}} \langle O_2 \rangle_0} \right|^2 . \tag{3.2}
$$

For $i \neq 7$, the contributions $\langle O_i \rangle_n$ are of order α_s. Therefore, to determine the order α_s corrections to M_n, it is consistent to take into account these contributions, and the α_s correction to the matrix element of the operator O_7 as well. But it is sufficient to know the Wilson coefficients C_i only to the presently available (leading log) accuracy.

The other important observation is that the two-loop part of the order α_s contributions to matrix elements of the operators at the low energy scale do not contribute to the moments of the photon spectrum. The reason is that these are virtual corrections that yield finite delta-function contributions at the maximal photon energy, and therefore they contribute equally to the numerator and the denominator of eq. (3.1). Thus, these contributions also cancel to order α_s in the moments M_n.

We mentioned earlier that, in the absence of the next-to-leading order result for the coefficient C_7^{eff}, any calculation of the photon spectrum (in particular that of [9]) is renormalization scheme dependent at order α_s. However, this scheme dependence affects again

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only the finite part of the delta-function at the maximal photon energy and therefore drops out from the moments.

These arguments prove that the moments M_n, as calculated below to order α_s, are renormalization scheme and scale independent.

IV. ORDER α_S QCD CORRECTIONS

As we mentioned in the introduction, the heavy quark expansion proves that the free quark decay model result is the leading term in a systematic expansion in powers of $1/m_b$, in which the first nonperturbative corrections arise at order $1/m_b^2$. In this section we discuss the order $\alpha_s(m_b)$ corrections to the moments of the photon spectrum in the free quark decay model. Following [9], we keep the strange quark mass finite to regularize collinear divergencies. We use the $\overline{\text{MS}}$ subtraction scheme, Feynman gauge, and dimensional regularization for infrared and ultraviolet divergencies, and phase-space integrals as well. We verified the calculation of Ref. [9], and also computed terms that were neglected in that calculation. The quantity that is simple to calculate in perturbation theory is

$$\delta m_n(x_0) = \frac{\displaystyle\int_{x_0}^{1} (x^n - 1) \, \frac{\mathrm{d}\Gamma_{\text{FQDM}}}{\mathrm{d}x} \, \mathrm{d}x}{\displaystyle\int_{x_0}^{1} \frac{\mathrm{d}\Gamma_{\text{FQDM}}}{\mathrm{d}x} \, \mathrm{d}x} \,, \tag{4.1}$$

where Γ_{FQDM} denotes the decay rate in free quark decay model, and we introduced the dimensionless parameters

$$x = \frac{2E_\gamma}{(1-r)\,m_b} \,, \qquad r = \frac{m_s^2}{m_b^2} \,. \tag{4.2}$$

The variable x corresponds to $E_\gamma/E_\gamma^{\max}$ in the free quark decay model. The definition (4.1) makes it apparent that the functions $\delta m_n(x_0)$ are proportional to α_s and that they are not affected by corrections proportional to $\delta(1-x)$ at this order. In the last section we shall discuss how to relate δm_n to the experimentally measurable moments M_n.

The conclusion of Ref. [9] is that near the maximal photon energy ($x = 1$) only the operators O_2 and O_7 are important. Although we found that the interference of O_7 and O_8

is also peaked near $x = 1$, this term turns out to be numerically small [10]. In this letter we only include the dominant contributions that were already calculated in [9], while we shall present the full order α_s calculation of the moments M_n elsewhere [10].

The contributions involving the operator O_2 are regular functions of the photon energy, and their explicit form can be found in [9]. The only singular (and numerically the most significant) contribution to the the free quark decay rate Γ_{FQDM} at order α_s comes from the operator O_7 alone. In the $r \to 0$ limit this contribution reads (we only explicitly present in this letter the corrections in the $r \to 0$ limit, but we included the r-dependent terms in our numerical results):

$$\frac{\mathrm{d}\Gamma_{77}}{\mathrm{d}x} = \Gamma_0 \left[\frac{m_b(\mu)}{m_b}\right]^2 \left\{\left[1 - \frac{\alpha_s C_F}{4\pi}\left(5 + \frac{4}{3}\pi^2 - 2\ln\frac{m_b^2}{\mu^2}\right)\right]\delta(1-x) \tag{4.3}$$
$$+ \frac{\alpha_s C_F}{4\pi}\left[7 + x - 2x^2 - 2(1+x)\ln(1-x) - \left(\frac{7}{1-x} + 4\frac{\ln(1-x)}{1-x}\right)_+\right]\right\},$$

where $C_F = 4/3$ in $SU(3)$, and

$$\Gamma_0 = \frac{G_F^2\,|V_{tb}V_{ts}^*|^2\,\alpha\,C_7^{\text{eff}}(\mu)^2}{32\,\pi^4}\,m_b^5\,. \tag{4.4}$$

By m_b we mean the b quark pole mass at order α_s, as discussed below. The $[f(x)]_+$ distribution corresponding to a function $f(x)$ acts on a test function $g(x)$ as

$$\int_0^1 [f(x)]_+\,g(x)\,\mathrm{d}x = \int_0^1 f(x)\,[g(x) - g(1)]\,\mathrm{d}x\,. \tag{4.5}$$

By including the μ-dependence in eq. (4.3), we can explicitly verify that Γ_{77} is μ-independent to order α_s. This cancellation by itself does not reduce significantly the μ-dependence of the theoretical prediction for the total decay rate, as the dominant part of $C_7^{\text{eff}}(\mu)$ arises from the mixing of O_2 with O_7 at order α_s, but still at leading log. The α_s correction in eq. (4.3) modifies the prediction for the total decay rate by about 15%, while it affects the first two moments of the photon spectrum, M_1 and M_2, by less than 3% and 5%, respectively. Our numerical results for δm_1 and δm_2, including the contribution of O_2, are shown in Table I.

When evaluating these corrections, it has to be kept in mind that the perturbative expansion becomes singular in the photon endpoint region, and a resummation of the per-turbative corrections may be required. As we are interested in the moments M_n for small n,

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and $(\alpha_s C_F/2\pi) \ln^2(1-x)$, the exponent of the Sudakov factor that suppresses the endpoint spectrum [11], becomes of order unity only around $x \sim 0.99$, our calculation is consistent without taking these effects into account.

V. NONPERTURBATIVE CORRECTIONS

We include the nonperturbative corrections to the free quark decay to order $1/m_b^2$ and leading order in α_s. At that order only corrections to the matrix element of the operator O_7 contribute. We present here the resulting corrections to the photon spectrum [12] only in the $r \to 0$ limit again:

$$\frac{d\Gamma}{dx} = \Gamma_0 \left(1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2}\right) \left[\delta(1-x) - \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \delta'(1-x) - \frac{\lambda_1}{6m_b^2} \delta''(1-x)\right]. \qquad (5.1)$$

The dimensionful constants λ_1 and λ_2 parameterize the matrix elements of the kinetic and chromomagnetic operators, respectively, which appear in the Lagrangian of the HQET at order $1/m_Q$:

$$\lambda_1 = \frac{1}{2} \langle M(v)| \, \bar{h}_v \, (iD)^2 \, h_v \, |M(v)\rangle \,,$$
$$\lambda_2 = \frac{1}{2d_M} \langle M(v)| \frac{g_s}{2} \, \bar{h}_v \, \sigma_{\alpha\beta} \, G^{\alpha\beta} \, h_v \, |M(v)\rangle \,, \qquad (5.2)$$

where $d_P = 3$ and $d_V = -1$ for pseudoscalar and vector mesons, respectively. $M(v)$ denotes the meson state and h_v denotes the quark field of the effective theory with velocity v. The numerical value of λ_2 can be extracted from the mass splitting between the vector and pseudoscalar mesons, $\lambda_2 = (m_{B^*}^2 - m_B^2)/4 \simeq 0.12 \, \text{GeV}^2$, while there is no similarly simple way to determine λ_1 from experiments (see, $e.g.$, [13]). Without including the order α_s corrections discussed in the previous section, one obtains to order $1/m_b^2$ in the heavy quark expansion

$$\langle E_\gamma \rangle = \frac{m_b}{2} \left(1 - \frac{\lambda_1 + 3\lambda_2}{2m_b^2}\right) = \frac{M_B - \bar{\Lambda}}{2} \,,$$
$$\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2 = -\frac{\lambda_1}{12} \,. \qquad (5.3)$$

In general, the central moments $\langle (E_\gamma - \langle E_\gamma \rangle)^n \rangle$ for $n \geq 2$ are proportional to $\lambda_1(m_b/2)^{n-2}$. Therefore, they are particularly useful for measuring λ_1. This is not unexpected, since λ_1 is the measure of the Fermi motion of the b quark that is responsible for the smearing of the photon spectrum. A comparison of the values of λ_1 extracted from different central moments can be used to estimate the systematic errors.

The parameter $\bar{\Lambda}$ in eq. (5.3) describes the mass difference between a heavy meson and the heavy quark that it contains, and it is one of the parameters that set the scale of the $1/m$ expansion [14]. It is related to the B meson mass and the b quark pole mass according to

$$M_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b}. \tag{5.4}$$

The quantity $\bar{\Lambda}$ suffers from renormalon ambiguities [15]. However, at any finite order in α_s it is consistent to extract $\bar{\Lambda}$ (or the pole mass m_b) from certain experiment(s), and use the resulting numerical values to evaluate theoretical predictions accurate to the same order in perturbation theory for other processes [16].

The series of the nonperturbative corrections to the moments of the photon spectrum, $M_n(E_0)$, is under control if the invariant mass of the final hadronic state corresponding to the lower cut E_0 is above the resonance region. Eq. (5.1) is only related to the experimentally measured spectrum once the theoretical expression is smeared over typical hadronic scales. This smearing is provided by taking the moments $M_n(E_0)$, if n is not too large, and E_0 is sufficiently far from $E_\gamma^{\max}$. Using the relation

$$\frac{M_B^2 - M_{X_s}^2}{2M_B} = E_\gamma, \tag{5.5}$$

we see that the present experimental signal region of $E_\gamma > 2.2\,\mathrm{GeV}$ [7] corresponds to $M_{X_s} \lesssim 2.2\,\mathrm{GeV}$. Even below this scale the widths of the X_s resonances are typically larger than their mass differences, and there are no resonances above $2.5\,\mathrm{GeV}$ [17]. It is still important for the reliability of our analysis to try to lower the experimental cut on the photon energy. For example, $E_0 = 2\,\mathrm{GeV}$ or $E_0 = 1.8\,\mathrm{GeV}$ would correspond to $M_{X_s} \lesssim 2.6\,\mathrm{GeV}$ or

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$M_{X_s} \lesssim 3\,\text{GeV}$, respectively. Varying E_0 provides a check on the systematic uncertainties: the extracted values of $\bar{\Lambda}$ and λ_1 should be unaffected by the variations of E_0, once the corresponding hadronic invariant mass is sufficiently above the resonance region.

VI. SUMMARY AND CONCLUSIONS

We can summarize our discussion by writing the theoretical prediction for the first two moments of the photon spectrum as

$$M_1(E_0) = \frac{M_B - \bar{\Lambda}}{2}\left[1 + \delta m_1\left(\frac{2E_0}{m_b}\right) + \mathcal{O}\left(\alpha_s^2,\, \alpha_s\frac{\Lambda^2}{m_b^2},\, \frac{\Lambda^3}{m_b^3}\right)\right],\qquad (6.1)$$

$$M_2(E_0) - M_1(E_0)^2 = -\frac{\lambda_1}{12} + \left(\frac{m_b}{2}\right)^2\left[\delta m_2\left(\frac{2E_0}{m_b}\right) - 2\,\delta m_1\left(\frac{2E_0}{m_b}\right) + \mathcal{O}\left(\alpha_s^2,\, \alpha_s\frac{\Lambda^2}{m_b^2},\, \frac{\Lambda^3}{m_b^3}\right)\right].$$

We used Λ to denote some QCD scale of order Λ_{QCD} or $\bar{\Lambda}$. To the order these relations are accurate, it is consistent to replace m_b by $M_B - \bar{\Lambda}$ everywhere in eq. (6.1). The numerical results for $\delta m_1(x_0)$ and $\delta m_2(x_0)$ are listed in Table I for three different values of x_0.

The significance of these relations is that they provide a reliable means of determining $\bar{\Lambda}$ (that is, $\bar{\Lambda}$ at order α_s [16]) and λ_1, or equivalently, measure the b quark pole mass at order α_s. Especially the first relation in eq. (6.1) is remarkable, since it is independent of λ_1 (and λ_2), and very sensitive to $\bar{\Lambda}$, with small theoretical uncertainties. The sensitivity of the second relation to λ_1 is reduced because of the factor $1/12$.

We would like to emphasize that the left-hand side of the relations in eq. (6.1) are measurable at CLEO; in fact, the central values can be extracted from Ref. [7]. As we do not know the cross-correlations of the errors on the data points, we are not in a position to quote numerical values for the experimental uncertainties. From the central values of the data, solving the first equation in (6.1), we find (with large uncertainties)

$$\bar{\Lambda} \sim 450\,\text{MeV}\,,\qquad m_b \sim 4.83\,\text{GeV}\,.\qquad (6.2)$$

We do not quote even a central value for λ_1, as the present experimental data do not constrain it to any reasonable accuracy. By varying all input parameters ($C_7(\mu)$ and $C_2(\mu)$

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corresponding to the range $m_b/2 < \mu < 2m_b$, α_s corresponding to $0.11 < \alpha_s(M_Z) < 0.13$, m_s between $100\,\mathrm{MeV}$ and $500\,\mathrm{MeV}$, m_t, and m_c) we find that the theoretical uncertainty of this measurement of $\bar{\Lambda}$ will be as small as $\pm 30\,\mathrm{MeV}$, while that of λ_1 about $\pm 0.15\,\mathrm{GeV}^2$.

In view of our earlier discussion, it is important to try to expand the experimental signal region. On the one hand, the systematic uncertainties inherent in our analysis (related to how well duality holds) can be estimated by varying the lower cut on the photon energy, as discussed at the end of Section V. On the other hand, expanding the signal region would diminish the sensitivity of the results as to whether the Sudakov logarithms at the endpoint are resummed or not.

The sensitivity of the moments of the spectrum to new physics is limited by how much operators other than O_7 affect M_n. We found that O_2 does not contribute to δm_1 and δm_2 by more than 10% in the SM. Given that the experimental constraint on the total decay rate from CLEO excludes large deviations from the SM, we conclude that the moments are largely insensitive to new physics. Even if physics beyond the SM contributes to the $B \to X_s \gamma$ decay, the proposed determination of $\bar{\Lambda}$ and λ_1 is likely to remain unaffected.

We conclude that the moments of the photon spectrum in the inclusive $B \to X_s \gamma$ decay will provide reliable measurements of fundamental parameters of QCD, which in turn will refine theoretical predictions for other observables in heavy quark decays. The complete order α_s calculation of the moments and a more detailed analysis of the theoretical uncertainties will be presented in a forthcoming paper [10].

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		$x_0 = 0.91$	$x_0 = 0.83$	$x_0 = 0.75$
$\delta m_1(x_0)$	$r = 4 \cdot 10^{-4}$	-0.014	-0.020	-0.025
	$r = 1 \cdot 10^{-2}$	-0.012	-0.017	-0.020
$\delta m_2(x_0)$	$r = 4 \cdot 10^{-4}$	-0.028	-0.040	-0.046
	$r = 1 \cdot 10^{-2}$	-0.023	-0.032	-0.038

TABLE I. Central values of $\delta m_1(x_0)$ and $\delta m_2(x_0)$ for two different values of m_s. $r = 4 \cdot 10^{-4}$ corresponds to $m_s = 100\,\mathrm{MeV}$, while $r = 1 \cdot 10^{-2}$ corresponds to a constituent quark mass $m_s = 500\,\mathrm{MeV}$. For $m_b \simeq 4.8\,\mathrm{GeV}$, $x_0 = 0.91$, $x_0 = 0.83$ and $x_0 = 0.75$ correspond to $E_0 = 2.2\,\mathrm{GeV}$, $E_0 = 2.0\,\mathrm{GeV}$ and $E_0 = 1.8\,\mathrm{GeV}$, respectively.